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VIBRATION OF SKEW PLATES AT LARGE AMPLITUDES INCLUDING SHEAR AND ROTATORY INERTIA EFFECTS

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Abstract—An approach to the large amplitude free, undamped flexural vibration of elastic, isotropic skew plates is developed with the aid of Hamilton's principle taking into consideration the effects of transverse shear and rotatory inertia. On the basis of an assumed vibration mode of the product form, the relationship between the amplitude and period is studied for skew plates of various aspect ratios and skew angles clamped along the boundaries. It is found that the time differential equation, i.e. modal equation when numerically integrated provides interesting information about the effects of transverse shear and rotatory inertia on aspect ratios and skew angles of thin and moderately thick skew plates both at small and at large amplitudes.

INTRODUCTION

The study of large amplitude flexural vibration of plates and shells has gained considerable importance in the recent years. The importance arises from the fact that when the flexural vibration involves large amplitudes the frequency of free or forced vibration is very much dependant upon the amplitude. The vibration of plates when the amplitude is large is governed by a system of coupled nonlinear differential equations as given by Herrmann[1]. Approximate solutions to these equations have been obtained by Chu and Herrmann[2], Yamaki[3] and several others. Since then several papers have appeared dealing with the large amplitude vibration of plates of various geometries[4]. Nowinski[5], using the stress function approach, developed the governing dynamic equations for skew plates and presented numerical results. Following this a study of large amplitude vibration of skew plates was carried out by the author[6, 7] wherein Berger approximation, influence of orthotropic parameters, inplane edge conditions, etc. were all investigated in detail.

Skew plates are extensively used in modern aircraft industry and the understanding of the vibration characteristics of such plates under severe operating conditions is becoming increasingly important. For the analysis of such class of problems many classical methods, however, are found to be inadequate since plate deflections during vibration are no longer small compared to the plate thickness. It is precisely this situation which introduces geometric nonlinearity necessitating the use of nonlinear strain displacement relations in the analysis. Consequently, the governing differential equations become nonlinear and difficult to deal with. Furthermore, the plate components may become moderately thick due to the actual design requirements involving the study of complicated effects such as transverse shear deformation and rotatory inertia on large amplitude vibration.

Reissner[8] extended the classical plate theory to take into account the effect of shear deformation on the static behaviour of plates and a further improvement in plate theory was later suggested to include the effects of shear deformation and rotatory inertia on dynamic behaviour of plates[9, 10]. Since then several papers dealing with these effects on plates of rectangular and circular geometries followed this approach. However, to the author's knowledge there seems to be no work reported on these lines for the dynamic analysis of skew plates. Attention is given in this paper to the study of the effects of geometric nonlinearity, transverse shear deformation and rotatory inertia on dynamic behaviour of skew plates.

A system of equations governing the large amplitude vibration of skew plates is derived to include the effects of transverse shear deformation and rotatory inertia. The solutions to the governing equations formulated in this paper are obtained using an assumed vibration mode and the Galerkin's method. Computations are restricted to the fundamental mode of the flexural vibration which is usually considered sufficient for practical or engineering purposes. Amplitude

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is plotted vs the period for different aspect ratios, skew angles and thickness length ratios of rectangular and skew plates taking into account the shear deformation and rotatory inertia effects. It is shown that these effects play a considerably important role in the dynamic behaviour of moderately thick plates.

GOVERNING EQUATIONS

The geometry of the plate and the coordinate system are shown in Fig. 1. x and y are the oblique coordinates and θ is the skew angle. ξ and η are the rectangular Cartesian coordinates. The material of the plate is assumed to be isotropic, elastic and homogeneous. Using u, v and w to represent the displacement components in the plane of the plate and transverse direction respectively, the strains of the median surface of the plate may be written as

$$\epsilon_1 = (\epsilon_x)_{z=0} = Cu_{,x} + Sv_{,x} + \frac{1}{2} w_{,x}^2$$

$$\epsilon_2 = (\epsilon_y)_{z=0} = v_{,y} + \frac{1}{2} w_{,y}^2$$

$$\gamma = (\gamma_{xy})_{z=0} = Cu_{,y} + Sv_{,y} + v_{,x} + w_{,x}w_{,y} \tag{1}$$

where

$$C = \cos\theta, \qquad S = \sin\theta.$$

Strains for any point away from the median surface at a distance z, where z is measured from the median surface normal to the plane of the plate are

$$\epsilon_x = \epsilon_1 + z\alpha_{,x}, \qquad \epsilon_y = \epsilon_2 + z\beta_{,y}, \qquad \epsilon_z = 0$$

$$\gamma_{xy} = \gamma + z(\alpha_{,y} + \beta_{,x}), \qquad \gamma_{xz} = \alpha + w_{,x}, \qquad \gamma_{yz} = \beta + w_{,y} \tag{2}$$

where α and β stand for the bending slope in the x and y directions, respectively.

In order to consider the effects of transverse shear deformation and rotatory inertia in the plate theory, the displacement components for a point off the median surface are taken in the following form

$$u(x, y, z, t) = Cu(x, y, t) + Sv(x, y, t) + z\alpha(x, y, t)$$

$$v(x, y, z, t) = v(x, y, t) + z\beta(x, y, t)$$

$$w(x, y, z, t) = w(x, y, t). \tag{3}$$

The stress–strain relations can be expressed as

$$\{\sigma_{ij}\} = [a_{ij}]\{\epsilon_{ij}\} \tag{4}$$

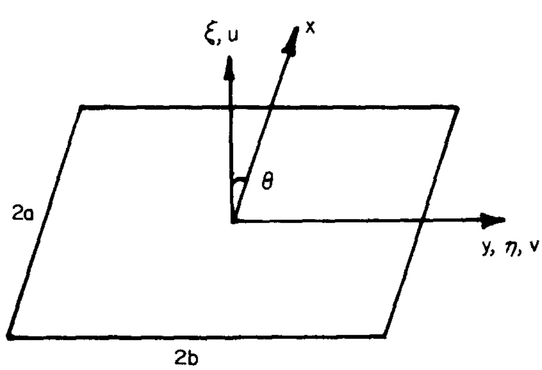

Fig. 1. Geometry and coordinate system.

or alternatively as

$$\{\epsilon_{ij}\} = [C_{ij}]\{\sigma_{ij}\} \tag{5}$$

where the coefficients a_{ij} and C_{ij} satisfy the reciprocal relations $a_{ij} = a_{ji}$, $C_{ij} = C_{ji}$ and are defined in the Appendix.

The stress and moment resultants, by definition, are

$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}\, dz, \qquad M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}\, z\, dz. \tag{6}$$

Thus,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = h \begin{bmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{41} & a_{42} & a_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma \end{Bmatrix} \tag{7}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{41} & a_{42} & a_{44} \end{bmatrix} \begin{Bmatrix} \alpha_{,x} \\ \beta_{,y} \\ \alpha_{,y} + \beta_{,x} \end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = h \begin{bmatrix} a_{55} & a_{56} \\ a_{65} & a_{66} \end{bmatrix} \begin{Bmatrix} \alpha + w_{,x} \\ \beta + w_{,y} \end{Bmatrix} \tag{9}$$

where, V_x and V_y are the transverse shear force per unit length of the plate and h is the thickness of the plate.

The strain energy of stretching of the middle surface, therefore, can be written as

$$U_s = \frac{h}{2} \int_x \int_y (a_{11}\epsilon_1^2 + a_{22}\epsilon_2^2 + a_{44}\gamma^2 + 2a_{12}\epsilon_1\epsilon_2 + 2a_{14}\epsilon_1\gamma + 2a_{24}\epsilon_2\gamma)\, dx\, dy. \tag{10}$$

The strain energy in bending becomes,

$$\begin{aligned} U_b = \int_x \int_y \Big\{ & M_x\alpha_{,x} + M_y\beta_{,y} + M_{xy}(\alpha_{,y} + \beta_{,x}) + V_x(\alpha + w_{,x}) + V_y(\beta + w_{,y}) \\ & - \frac{6}{h^3}(C_{11}M_x^2 + C_{22}M_y^2 + C_{44}M_{xy}^2 + 2C_{12}M_xM_y + 2C_{14}M_xM_{xy} + 2C_{24}M_yM_{xy}) \\ & - \frac{3}{5h}(C_{55}V_x^2 + C_{66}V_y^2 + 2C_{56}V_xV_y) \Big\}\, dx\, dy. \end{aligned} \tag{11}$$

It is to be noted that the above expression for the bending strain energy includes the effect of transverse shear deformation. If this effect is neglected in the analysis eqn (11) can be shown to reduce to eqn (7) of Ref. [6].

The expression for the kinetic energy of the plate can be written as

$$T = \frac{C\rho}{2} \int_x \int_y \left\{ h(u_{,t}^2 + v_{,t}^2 + w_{,t}^2) + \frac{h^3}{12}(\alpha_{,t}^2 + \beta_{,t}^2) \right\} dx\, dy \tag{12}$$

where ρ is the mass per unit volume of the plate. With the aid of eqns (10)–(12) and with the use of Hamilton's principle along with Reissner's variational theorem, the governing equations of motion are derived and are given below in a considerably simplified form

$$N_{x,x} + N_{xy,y} = 0 \tag{13}$$

$$N_{y,y} + N_{xy,x} = 0 \tag{14}$$

$$V_{x,x} + V_{y,y} + N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} = C\rho h w_{,tt} \tag{15}$$

$$M_{x,x} + M_{xy,y} - V_x - \frac{C\rho h^3}{12} \alpha_{,tt} = 0 \tag{16}$$

$$M_{y,y} + M_{xy,x} - V_y - \frac{C\rho h^3}{12} \beta_{,tt} = 0 \tag{17}$$

$$\alpha_{,x} - \frac{12}{h^3}(C_{11}M_x + C_{12}M_y + C_{14}M_{xy}) = 0 \tag{18}$$

$$\beta_{,y} - \frac{12}{h^3}(C_{22}M_y + C_{12}M_x + C_{24}M_{xy}) = 0 \tag{19}$$

$$\alpha_{,y} + \beta_{,x} - \frac{12}{h^3}(C_{14}M_x + C_{24}M_y + C_{44}M_{xy}) = 0 \tag{20}$$

$$\alpha + w_{,x} - \frac{6}{5h}(C_{55}V_x + C_{56}V_y) = 0 \tag{21}$$

$$\beta + w_{,y} - \frac{6}{5h}(C_{66}V_y + C_{56}V_x) = 0. \tag{22}$$

It can be seen from the governing eqns (13)–(22) that they take care of the effects of transverse shear deformation and rotatory inertia.

The two in-plane equilibrium equations given by eqns (13) and (14) are automatically satisfied by the stress function $F(x, y)$ defined as

$$N_x = hF_{,yy}, \qquad N_y = hF_{,xx}, \qquad N_{xy} = -hF_{,xy}. \tag{23}$$

Solving for V_x and V_y from eqns (21) and (22) and upon substitution of V_x and V_y and eqn (23), eqn (15) becomes

$$I + D_{17}(\beta_{,x} + \alpha_{,y} + 2w_{,xy}) - D_{19}(\alpha_{,x} + w_{,xx}) - D_{18}(\beta_{,y} + w_{,yy}) - C\rho h w_{,tt} = 0 \tag{24}$$

where

$$I = h(F_{,yy}w_{,xx} + F_{,xx}w_{,yy} - 2F_{,xy}w_{,xy}).$$

The remaining seven equations namely eqns (16)–(22) can be reduced to two equations in terms of α, β and w as given below:

$$\alpha + w_{,x} - D_1\alpha_{,xx} - D_2\beta_{,yy} - D_3\alpha_{,yy} - D_4\beta_{,xx} - D_5\beta_{,xy} - D_6\alpha_{,xy} + D_7\alpha_{,tt} + D_8\beta_{,tt} = 0 \tag{25}$$

$$\beta + w_{,y} - D_9\alpha_{,xx} - D_{10}\beta_{,yy} - D_{11}\alpha_{,yy} - D_{12}\beta_{,xx} - D_{13}\beta_{,xy} - D_{14}\alpha_{,xy} + D_{15}\beta_{,tt} + D_{16}\alpha_{,tt} = 0 \tag{26}$$

where the coefficients $D_1 - D_{19}$ are defined in the Appendix. The compatibility condition in terms of the stress function F, becomes [6]

$$F_{,xxxx} + K_1F_{,yyyy} + K_2F_{,xxyy} + K_3F_{,xxxy} + K_4F_{,xyyy} = CE(w_{,xy}^2 - w_{,xx}w_{,yy}) \tag{27}$$

where $K_1 - K_4$ are defined in the Appendix.

Equations (24)–(27) are then the governing equations for the large amplitude free, undamped flexural vibration of isotropic skew plates which take into account the effects of geometric nonlinearity, transverse shear deformation and rotatory inertia. Equations (25) and (26) can be solved simultaneously for α and β thus eliminating these from eqn (24).

ILLUSTRATION

The set of equations given by eqns (24)–(27) are coupled and nonlinear in nature and exact solution to these is difficult to obtain. Therefore the theory is illustrated here by application to the free, undamped vibration of a skew plate clamped along the boundaries where only an approximate solution is attempted. For a plate of dimensions $2a$, $2b$ an expression in the following form is assumed in order to satisfy the boundary conditions

$$\frac{w}{h}=\frac{f(\tau)}{4}\left(1+\cos\frac{\pi x}{a}\right)\left(1+\cos\frac{\pi y}{b}\right). \tag{28}$$

Assuming that the clamping of the edges of the plate does not prevent free in-plane motions[5], the stress function F is obtained as follows by solving eqn (27).

$$\begin{aligned} F(x,y,\tau) &= a_1\cos\frac{\pi x}{a}+a_2\cos\frac{\pi y}{b}+a_3\cos\frac{2\pi x}{a}+a_4\cos\frac{2\pi y}{b} \\ &+a_5\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}+a_6\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}+a_7\cos\frac{\pi y}{b}\cos\frac{2\pi x}{a}+a_8\sin\frac{\pi x}{a}\sin\frac{2\pi y}{b} \\ &+a_9\sin\frac{\pi y}{b}\sin\frac{2\pi x}{a}+a_{10}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \end{aligned} \tag{29}$$

where the coefficients a_1-a_{10} are given in the Appendix. Let

$$I_1 = I+2D_{17}w_{,xy}-D_{18}w_{,yy}-D_{19}w_{,xx}-C\rho h w_{,tt} \tag{30}$$

$$J = 1-D_1\frac{\partial^2}{\partial x^2}-D_3\frac{\partial^2}{\partial y^2}-D_6\frac{\partial^2}{\partial x\partial y}+D_7\frac{\partial^2}{\partial t^2} \tag{31}$$

$$K = D_8\frac{\partial^2}{\partial t^2}-D_2\frac{\partial^2}{\partial y^2}-D_4\frac{\partial^2}{\partial x^2}-D_5\frac{\partial^2}{\partial x\partial y} \tag{32}$$

$$L = D_{16}\frac{\partial^2}{\partial t^2}-D_9\frac{\partial^2}{\partial x^2}-D_{11}\frac{\partial^2}{\partial y^2}-D_{14}\frac{\partial^2}{\partial x\partial y} \tag{33}$$

$$M = 1-D_{10}\frac{\partial^2}{\partial y^2}-D_{12}\frac{\partial^2}{\partial x^2}-D_{13}\frac{\partial^2}{\partial x\partial y}+D_{15}\frac{\partial^2}{\partial t^2} \tag{34}$$

$$\begin{aligned} N &= D_{20}\frac{\partial^4}{\partial x^4}+D_{21}\frac{\partial^4}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4}{\partial y^4}+D_{23}\frac{\partial^4}{\partial x^3\partial y} \\ &+D_{24}\frac{\partial^4}{\partial x\partial y^3}+D_{25}\frac{\partial^4}{\partial t^4}+D_{26}\frac{\partial^4}{\partial x^2\partial t^2}+D_{27}\frac{\partial^4}{\partial x\partial y\partial t^2} \\ &+D_{28}\frac{\partial^4}{\partial y^2\partial t^2}+D_{29}\frac{\partial^2}{\partial x^2}+D_{30}\frac{\partial^2}{\partial y^2}+D_{31}\frac{\partial^2}{\partial x\partial y}-D_{32}\frac{\partial^2}{\partial t^2}-1 \end{aligned} \tag{35}$$

$$\begin{aligned} R &= 2D_{17}\frac{\partial^2}{\partial x\partial y}-D_{19}\frac{\partial^2}{\partial x^2}-D_{18}\frac{\partial^2}{\partial y^2}+D_{33}\frac{\partial^4}{\partial x^4} \\ &+D_{34}\frac{\partial^4}{\partial y^4}+D_{35}\frac{\partial^4}{\partial x^2\partial y^2}+D_{36}\frac{\partial^4}{\partial x\partial y^3}+D_{37}\frac{\partial^4}{\partial x^3\partial y} \\ &+D_{38}\frac{\partial^4}{\partial x^2\partial t^2}+D_{39}\frac{\partial^4}{\partial y^2\partial t^2}+D_{40}\frac{\partial^4}{\partial x\partial y\partial t^2} \end{aligned} \tag{36}$$

where the coefficients $D_{20}-D_{40}$ are defined in the Appendix. Equation (24) can now be rewritten as

$$I_1+D_{17}(\alpha_{,y}+\beta_{,x})-D_{18}\beta_{,y}-D_{19}\alpha_{,x}=0. \tag{37}$$

From eqns (25) and (26) it follows that

$$N(\alpha)=M(w_{,x})-K(w_{,y}) \tag{38}$$

$$N(\beta)=J(w_{,y})-L(w_{,x}). \tag{39}$$

Using the operator N on eqn (37) and substituting for $N(\alpha)$, $N(\beta)$ from eqns (38) and (39) it follows that

$$N(I_1) + R(w) = 0. \tag{40}$$

Thus eqns (24)–(26) are reduced to one equation, namely eqn (40).

Substituting the assumed mode shape w from eqn (28) and F from eqn (29), eqn (40) is solved approximately using Galerkin's method. The result, after considerable simplification is a time differential equation in f as given below:

$$\begin{aligned} &C_1 f + C_2 \ddot{f} + C_3 \ddddot{f} + C_4 \overset{\ddot{\cdots}}{f} + C_5 f^3 + C_6 f^2 \ddot{f} + C_7 f(\dot{f})^2 \\ &C_8 f \dot{f} \ddot{f} + C_9 f(\ddot{f})^2 + C_{10}(\dot{f})^2 \ddot{f} + C_{11} f^2 \ddddot{f} = 0 \end{aligned} \tag{41}$$

where $(\dot{\ }) = (\partial/\partial\tau)$, $\tau = tq^{(1/2)}$ and the coefficients $C_1 - C_{11}$ are defined in the Appendix.

Equation (41) is the modal equation applicable for the large amplitude free, undamped flexural vibration of a clamped skew plate with movable edges. This ordinary nonlinear differential equation is solved numerically using the Runge–Kutta method. During the evaluation of the coefficients $C_1 - C_{11}$, it was observed that C_3, C_4, C_8, C_9, C_{10} and C_{11} were neglibible in comparison to others and therefore the terms containing these coefficients have been thrown out in the final solution of eqn (41). With this simplification in mind, eqn (41) has been numerically integrated taking the time interval $\nabla\tau$ as 0.001. The ratio of the nonlinear period of vibration T, including the effects of transverse shear deformation and rotatory inertia, to the corresponding linear period T_0 of a classical plate, not including these effects, has been computed for different nondimensional amplitudes, plate aspect ratios, skew angles and thickness–length ratios of the plate. The results are presented in graphical form in Fig. 2–13.

CONCLUDING REMARKS

A set of equations governing the large amplitude free, undamped flexural vibration of skew plates to include the effects of transverse shear deformation and rotatory inertia are derived in

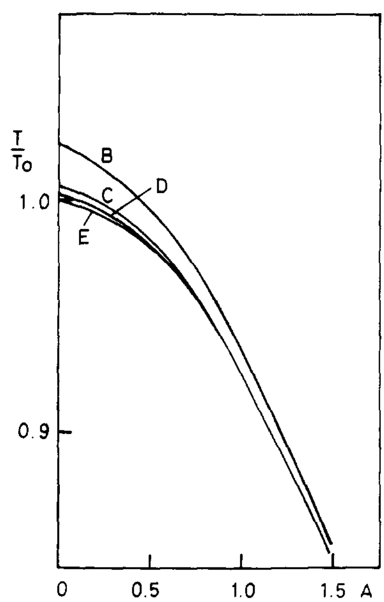

Fig. 2.

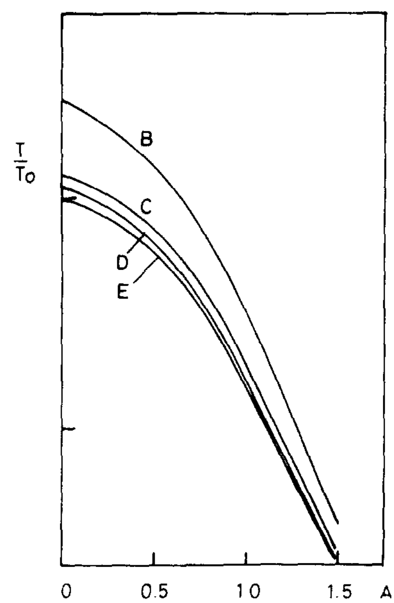

Fig. 3.

Fig. 2. T/T_0 vs amplitude $\theta = 0°$, $r = 0.5$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

Fig. 3. T/T_0 vs amplitude $\theta = 0°$, $r = 1.0$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

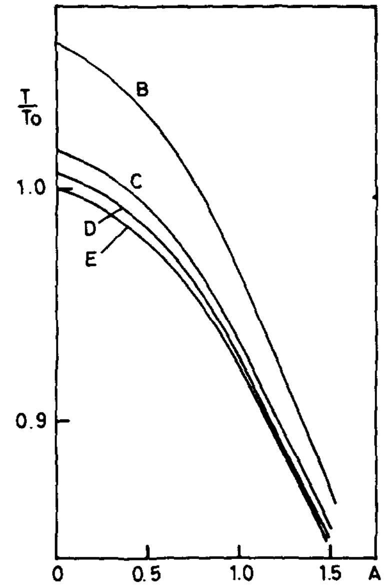

Fig. 4.

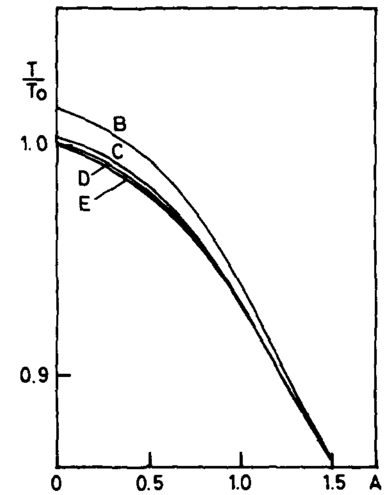

Fig. 5.

Fig. 4. T/T_0 vs amplitude $\theta = 0°$, $r = 1.5$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

Fig. 5. T/T_0 vs amplitude $\theta = 15°$, $r = 0.5$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

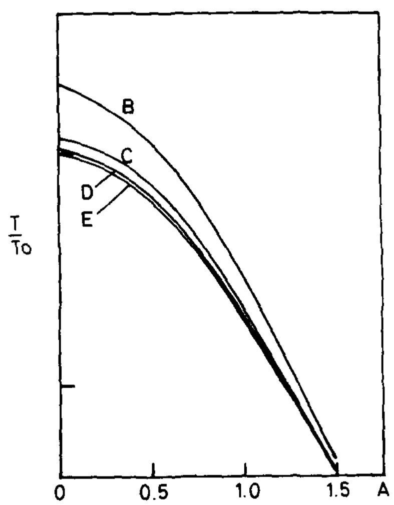

Fig. 6.

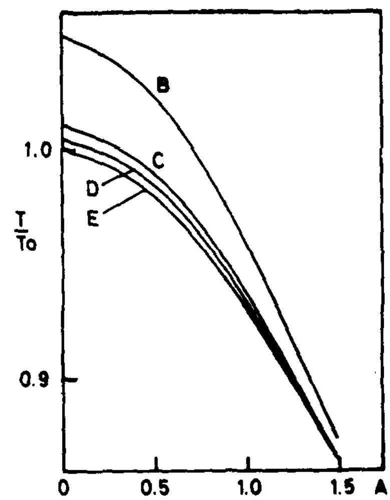

Fig. 7.

Fig. 6. T/T_0 vs amplitude $\theta = 15°$, $r = 1.0$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

Fig. 7. T/T_0 vs amplitude $\theta = 15°$, $r = 1.5$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

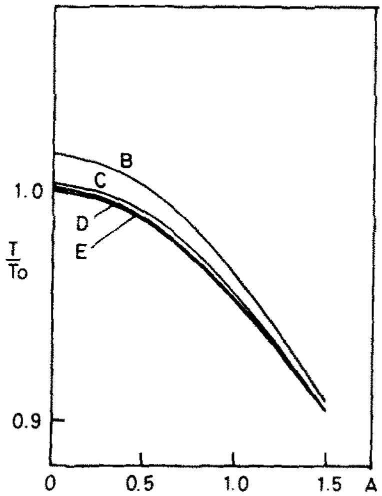

Fig. 8. T/T_0 vs amplitude $\theta = 30°$, $r = 0.5$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

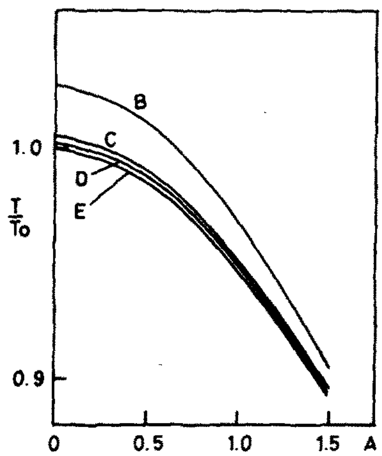

Fig. 9. T/T_0 vs amplitude $\theta = 30°$, $r = 1.0$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

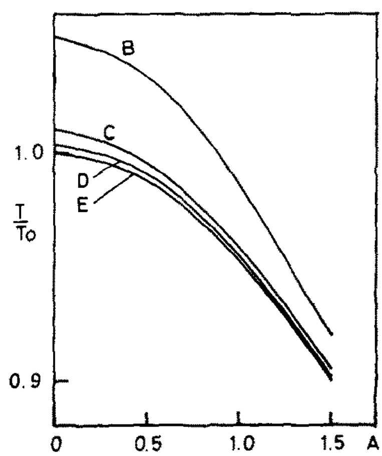

Fig. 10. T/T_0 vs amplitude $\theta = 30°$, $r = 1.5$; B, $(h/2a) = (1/10)$; C, $(h/2a) = (1/20)$; D, $(h/2a) = (1/30)$; E, no transverse shear or rotatory inertia.

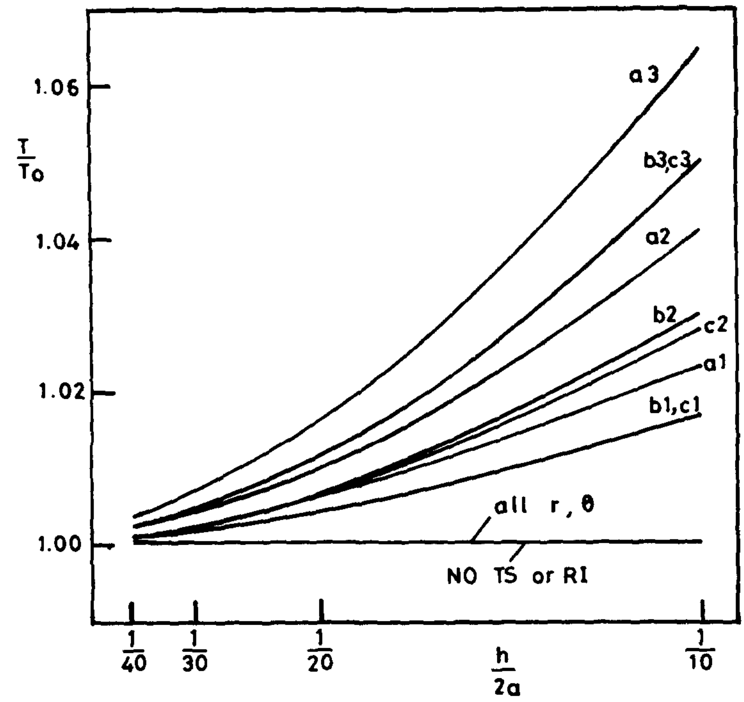

Fig. 11. T/T_0 vs $(h/2a)$, amplitude = 0; a, $\theta = 0°$; b, $\theta = 15°$; c, $\theta = 30°$; 1, $r = 0.5$; 2, $r = 1.0$; 3, $r = 1.5$

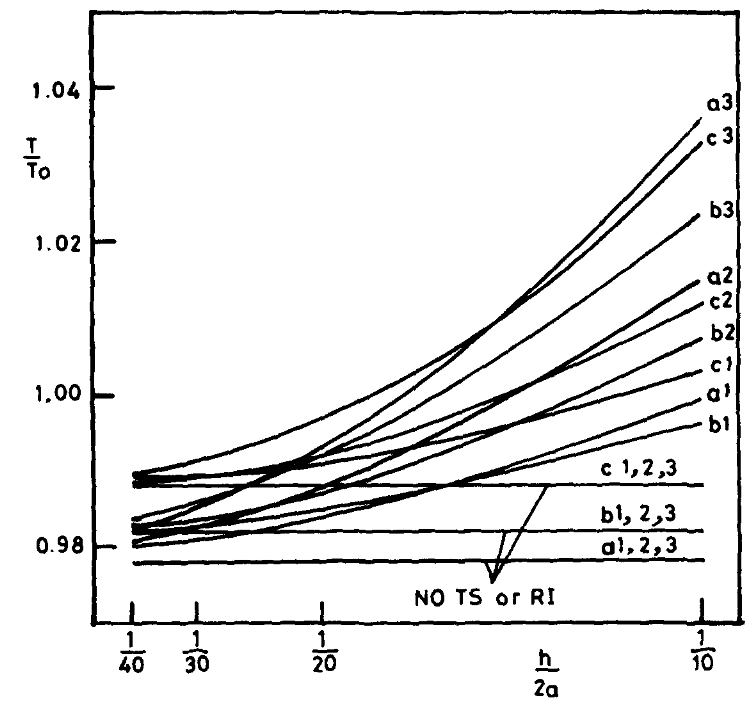

Fig. 12. T/T_0 vs $(h/2a)$, amplitude = 0.5; a, $\theta = 0°$; b, $\theta = 15°$; c, $\theta = 30°$; 1, $r = 0.5$; 2, $r = 1.0$; 3, $r = 1.5$.

this paper using the stress function approach. The governing equations and the numerical results reported in this paper are in excellent agreement with those in the literature so far available only for rectangular plates. Amplitude is plotted vs period for different aspect ratios, skew angles and thickness length ratios of plates. For the sake of easy comparison results where these effects are not considered are also plotted. The influence of transverse shear and rotatory inertia on large amplitude vibration of plates in general is shown by a consistent

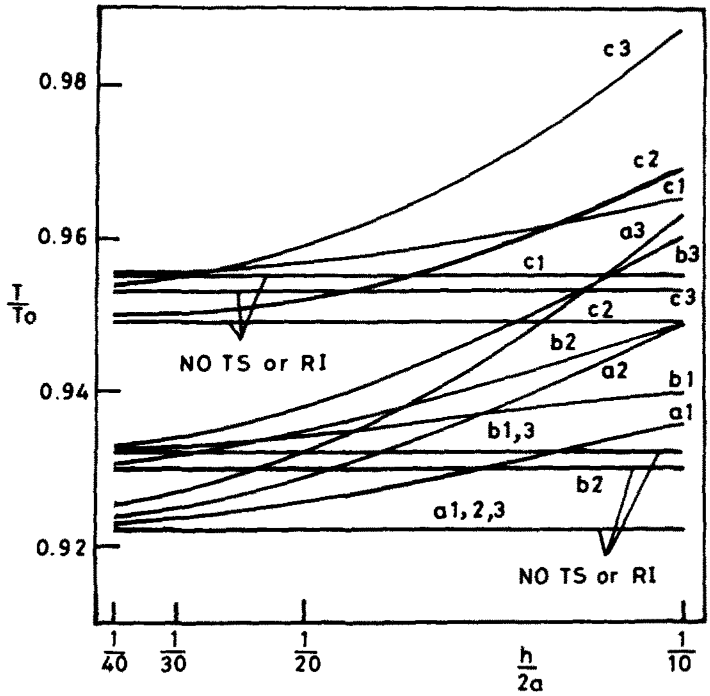

Fig. 13. T/T_0 vs $(h/2a)$, amplitude = 1.0; a, $\theta = 0°$; b, $\theta = 15°$; c, $\theta = 30°$; 1, $r = 0.5$; 2, $r = 1.0$; 3, $r = 1.5$.

increase in the nonlinear period although the increase is less at moderately large amplitudes. It is seen that these effects play a considerably important role in the period-amplitude behaviour of moderately thick plates and have practically no influence on thin plates with thickness-length ratio less than 0.05. Rectangular plates seem to respond more to these effects at relatively small amplitudes whereas at moderately large amplitudes the trend reverses and skew plates are influenced more by these effects.

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APPENDIX

$$a_{11} = \frac{E_1}{C^3},\quad a_{22} = C\{2t_1^2(E_3 + 2G) + E_1(1 + t_1^4)\},$$

$$a_{44} = \frac{1}{C}(G + E_1 t_1^2),\quad a_{55} = \frac{G}{C},\quad a_{66} = a_{55},\quad a_{12} = \frac{1}{C}(E_3 + E_1 t_1^2),$$

$$a_{14} = -\frac{E_1 t_1}{C^2},\quad a_{24} = -t_1(2G + E_1 t_1^2 + E_3),\quad a_{56} = -Gt_1,$$

$$a_{ij} = a_{ji},\quad t_1 = S/C,\quad a_{i3} = a_{i5} = a_{i6} = 0;\qquad i = 1, 2, 3, 4.$$

$$e_0 = a_{14}a_{22} - a_{24}a_{12},\quad e_1 = a_{24}a_{14} - a_{44}a_{12},$$

$$e_2 = a_{55}a_{66} - a_{56}^2,\quad e_3 = a_{12}^2 - a_{11}a_{22},$$
$$e_4 = a_{12}a_{14} - a_{11}a_{24},\quad e_5 = e_0e_4 - e_1e_3,$$
$$C_{11} = \frac{1}{a_{11}}(1 - a_{12}C_{12} - a_{14}C_{14}),\quad C_{22} = \frac{1}{a_{22}}(1 - a_{12}C_{12} - a_{24}C_{24}),$$
$$C_{44} = \frac{1}{a_{44}}(1 - a_{14}C_{14} - a_{24}C_{24}),\quad C_{55} = \frac{1}{a_{55}}(1 - a_{56}C_{56}),$$
$$C_{66} = \frac{a_{55}}{e_2},\quad C_{12} = \frac{1}{e_3}(a_{12} - e_4C_{14}),\quad C_{14} = \frac{1}{e_5}e_0a_{12},$$
$$C_{24} = \frac{1}{e_0}(e_3C_{12} - a_{12}),\quad C_{56} = \frac{1}{a_{56}}(1 - a_{66}C_{66}),$$
$$C_{i3} = C_{i5} = C_{i6} = 0,\quad i = 1,2,3,4, E_1 = \frac{E}{1-\nu^2},\quad E_3 = E_1\nu$$

E, ν, G are the material constants of the isotropic plate.

$$D_1 = H(C_{55}G_1 + C_{56}G_2),\quad D_2 = H(C_{56}G_3 - C_{55}G_4),$$
$$D_3 = H(C_{55}G_5 - C_{56}G_6),\quad D_4 = H(C_{55}G_7 + C_{56}G_5),$$
$$D_5 = H(C_{55}G_8 - C_{56}G_9),\quad D_6 = H(C_{55}G_{10} - C_{56}G_{11}),$$
$$D_7 = G_{12}C_{55},\quad D_8 = G_{12}C_{56},\quad D_9 = H(C_{56}G_1 + C_{66}G_2),$$
$$D_{10} = H(C_{66}G_3 - C_{56}G_4),\quad D_{11} = H(C_{56}G_5 - C_{66}G_6),$$
$$D_{12} = H(C_{56}G_7 + C_{66}G_5),\quad D_{13} = H(C_{56}G_8 - C_{66}G_9),$$
$$D_{14} = H(C_{56}G_{10} - C_{66}G_{11}),\quad D_{15} = G_{12}C_{66},\quad D_{16} = D_8,$$
$$H = \frac{h^2}{10C''},\quad G_1 = \frac{B_1}{C_{11}},\quad G_2 = C_3'C_{12},\quad G_3 = B_4,\quad G_4 = B_5,$$
$$G_5 = C_1'C_{12},\quad G_6 = C_2'C_{12},\quad G_7 = \frac{B_3}{C_{11}},\quad G_8 = \frac{B_2}{C_{11}} + G_5,$$
$$G_9 = G_4 + G_6,\quad G_{10} = \frac{B_3}{C_{11}} + G_2,\quad G_{11} = (C_4' - C_1')C_{12},$$
$$G_{12} = \frac{C\rho h^2}{10},\quad B_1 = C' + C_4'C_{12}^2 - C_3'C_{12}C_{14},$$
$$B_2 = C_{14}B_5 - C_{12}B_4,\quad B_3 = C_{12}(C_2'C_{12} - C_1'C_{14}),$$
$$B_4 = C_4'C_{11} + C_2'C_{14},\quad B_5 = C_3'C_{11} + C_1'C_{14},$$
$$C_1' = C_{11}C_{22} - C_{12}^2,\quad C_2' = C_{11}C_{24} - C_{12}C_{14},$$
$$C_3' = C_{12}C_{24} - C_{14}C_{22},\quad C_4' = C_{44}C_{12} - C_{14}C_{24},$$
$$C' = C_1'C_4' - C_2'C_3',\quad C'' = C_{56}^2 - C_{55}C_{66},\quad H_1 = \frac{5h}{6C''}$$
$$D_{17} = H_1C_{56},\quad D_{18} = H_1C_{55},\quad D_{19} = H_1C_{66},\quad D_{20} = D_4D_9 - D_1D_{12},$$
$$D_{21} = D_2D_9 + D_4D_{11} + D_5D_{14} - D_1D_{10} - D_3D_{12} - D_6D_{13},$$
$$D_{22} = D_2D_{11} - D_3D_{10},\quad D_{23} = D_4D_{14} + D_5D_9 - D_1D_{13} - D_6D_{12},$$
$$D_{24} = D_2D_{14} + D_5D_{11} - D_3D_{13} - D_6D_{10},\quad D_{25} = D_8D_{16} - D_7D_{15},$$
$$D_{26} = D_1D_{15} + D_7D_{12} - D_8D_9 - D_4D_{16},\quad D_{27} = D_6D_{15} + D_7D_{13} -$$
$$D_8D_{14} - D_5D_{16},\quad D_{28} = D_3D_{15} + D_7D_{10} - D_8D_{11} - D_2D_{16},$$
$$D_{29} = D_1 + D_{12},\quad D_{30} = D_3 + D_{10},\quad D_{31} = D_6 + D_{13},\quad D_{32} = D_7 + D_{15},$$
$$D_{33} = D_{17}D_9 + D_{19}D_{12},\quad D_{34} = D_{17}D_2 + D_{18}D_3,$$
$$D_{35} = D_{17}(D_{11} + D_4 - D_{13} - D_6) - D_{19}(D_5 - D_{10}) - D_{18}(D_{14} - D_1),$$
$$D_{36} = D_{17}(D_5 - D_{10} - D_3) - D_2D_{19} + D_{18}(D_6 - D_{11}),$$
$$D_{37} = D_{17}(D_{14} - D_{12} - D_1) + D_{19}(D_{13} - D_4) - D_{18}D_9,$$
$$D_{38} = -(D_{17}D_{16} + D_{19}D_{15}),\quad D_{39} = -(D_{17}D_8 + D_{18}D_7),$$
$$D_{40} = D_{17}(D_{15} + D_7) + D_{19}D_8 + D_{18}D_{16}$$
$$K_1 = C^4(1 + t_1^2)^2,\quad K_2 = 2(1 + 2S^2),\quad K_3 = K_4 = -4S,$$
$$d = \frac{-EC(fhr)^2}{16},\quad r = \frac{a}{b},\quad a_1 = \frac{d}{2},\quad a_2 = \frac{a_1}{K_1r^4},$$
$$a_3 = \frac{a_1}{16},\quad a_4 = \frac{a_2}{16},\quad a_5 = \frac{2a_1m_1}{m_3},\quad a_6 = \frac{a_1m_7}{m_9},\quad a_7 = \frac{a_1m_4}{m_6},$$

$$a_8 = \frac{a_1 m_8}{m_9}, \quad a_9 = \frac{a_1 m_5}{m_6}, \quad a_{10} = \frac{2a_1 m_2}{m_3}, \quad A_{11} = \frac{1}{2}\left(a_2 + a_5 + \frac{a_7}{2}\right),$$

$$A_{12} = \frac{1}{2}\left(a_1 + a_5 + \frac{a_6}{2}\right), \quad A_{13} = a_1 + a_2 + 2(a_3 + a_4) + \frac{1}{2}(a_6 + a_7),$$

$$A_{14} = a_7 + 4a_3, \quad A_{15} = a_6 + 4a_4, \quad A_{16} = \frac{1}{2}(a_8 + a_9),$$

$$m_1 = 1 + r^2(K_2 + K_1 r^2), \quad m_2 = r(K_3 + K_4 r^2), \quad m_3 = m_1^2 - m_2^2,$$

$$m_4 = 16 + r^2(4K_2 + K_1 r^2), \quad m_5 = 2r(4K_3 + K_4 r^2), \quad m_6 = m_4^2 - m_5^2,$$

$$m_7 = 1 + 4r^2(K_2 + 4K_1 r^2), \quad m_8 = 2r(K_3 + 4K_4 r^2), \quad m_9 = m_7^2 - m_8^2,$$

$$q = \frac{E}{\rho a^2}, \quad m = \frac{\pi^2}{a^2}, \quad n = \frac{\pi^2}{b^2}, \quad e = \frac{C\rho h^2}{4}, \quad C_1 = g_{22},$$

$$C_2 = g_{23} q, \quad C_3 = g_{24} q^2, \quad C_4 = g_{15} q^3, \quad C_5 = d_5,$$

$$C_6 = q(d_7 + d_8), \quad C_7 = 2C_6, \quad C_8 = 8d_6 q^2, \quad C_9 = \frac{3}{4} C_8,$$

$$C_{10} = 2C_9, \quad C_{11} = \frac{C_8}{8},$$

$$d_1 = m^2\{2A_{11}D_{20} + 2A_{12}D_{22}r^4 + (D_{20} + D_{21}r^2 + D_{22}r^4)A_{13} - rA_{16}(D_{23} + D_{24}r^2)\}$$

$$d_2 = m\{A_{16}D_{31}r - 2A_{11}D_{29} - 2A_{12}D_{30}r^2 - A_{13}(D_{29} + D_{30}r^2)\}$$

$$d_3 = 2(A_{11} + A_{12}) + A_{13}, \quad d_4 = m\{2A_{11}D_{26} + 2A_{12}D_{28}r^2 - A_{16}D_{27}r + A_{13}(D_{26} + D_{28}r^2)\},$$

$$d_5 = d_9(d_1 + d_2 - d_3), \quad d_6 = 3D_{25}d_3 d_9,$$

$$d_7 = -3D_{32}d_3 d_9, \quad d_8 = -3d_4 d_9, \quad d_9 = \frac{mnh^2}{4}$$

$$g_1 = D_{20}m^2 - D_{29}m - 1, \quad g_2 = n(D_{22}n - D_{30}),$$

$$g_3 = mr(D_{23}m + D_{24}n - D_{31}), \quad g_4 = D_{26}m, \quad g_5 = D_{28}n,$$

$$g_6 = D_{27}mr, \quad g_7 = (D_{19} + D_{18}r^2), \quad g_8 = g_1 + g_2 + g_{11},$$

$$g_9 = 3\{m(D_{19} + D_{33}m) + n(D_{18} + D_{34}n)\} + D_{35}mn,$$

$$g_{10} = -(D_{38}m + D_{39}n), \quad g_{11} = D_{21}mn,$$

$$g_{12} = \frac{mh}{4}\{g_7 g_8 + 2D_{19}g_1 + 2D_{18}r^2(g_2 - 1) - 2D_{17}rg_3\},$$

$$g_{13} = e\{-4 + 2g_1 + 2(g_2 - 1) + g_8\}, \quad g_{14} = 9eD_{32},$$

$$g_{15} = -9eD_{25}, \quad g_{16} = -\frac{3}{4} mhD_{32}g_7,$$

$$g_{17} = -\frac{mh}{4}\{2D_{19}g_4 + 2D_{18}r^2 g_5 + (g_4 + g_5)g_7 - 2D_{17}rg_6\},$$

$$g_{18} = \frac{3}{4} D_{25}mh\, g_7, \quad g_{19} = \frac{h}{4} g_9, \quad g_{20} = \frac{3}{4} h\, g_{10},$$

$$g_{21} = 3e(g_4 + g_5), \quad g_{22} = g_{12} + g_{19},$$

$$g_{23} = g_{13} + g_{16} + g_{17} + g_{20},$$

$$g_{24} = g_{14} + g_{18} + g_{21}.$$